Capacitor circuits

# Aims

solve problems using formulae for capacitors in series and in parallel.

deduce, from the area under a potential-charge graph, the equation *W* = ½*QV* and hence *W* = ½*CV*2.

Capacitors in parallel

The aim is to find one capacitor, C, to replace C1 and C2.

+Q -Q

+Q2 -Q2

+Q1 -Q1

V

V

C

# C1

# C2

In this case, the pds across the capacitors must be the same so the charge stored on each will be different and we have:

Q1 =

Q2 =

and

Q =

For the two circuits to be equivalent,

Q =

C x V =

i.e. **C = C1 + C2**

which can be extended to: **C = C1 + C2 + C3 + ……**

Capacitors in series

Again, the aim is to find a capacitor C which has the same effect as capacitors C1 and C2. This means that for the same potential difference V the charge stored must be the same.

+Q -Q

+Q -Q

+Q -Q

V

V

C1

C2

C

If either the left hand plate of C1 or the right hand plate of C2 held an excess of charge then electrons would move until there was equal charge on the two plates. (Another way to think of this is that the section consisting of the right hand plate of C1 and the left-hand plate of C2 is isolated and uncharged.) So:

* the pd across C1 is V1 =
* the pd across C2 is V2 =
* and the pd across C is V =

but for components in series:

V =

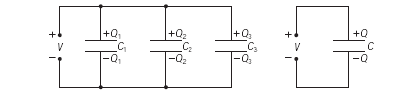
So Q/C =

i.e. **1/C = 1/C1 + 1/C2**

which for more capacitors can be extended to: **1/C = 1/C1 + 1/C2 + 1/C3 ……**

Capacitors in series and parallel questions

The diagram below shows three capacitors of capacitance *C*1, *C*2 and *C*3 connected in parallel to a potential difference *V.* Work through question 1 in order to obtain the value, in terms of *C*1, *C*2 and *C*3, of a single capacitor *C* that would replace all three.



Question

1. (a) Write down expressions for the following:

(i) *C*1 in terms of *V* and *Q*1

(ii) *C*2 in terms of *V* and *Q*2

(iii) *C*3 in terms of *V* and *Q*

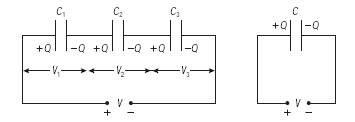
(iv) *C* in terms of *V* and *Q*3

(b) What is the total charge *Q* of all three capacitors in terms of *Q*1, *Q*2 and *Q*3?

(c) Using your answers to (a) and (b), obtain an expression for *C* in terms of *C*1, *C*2 and *C*3.

Capacitors in series

Three capacitors C1, C2 and C3 connected in series are shown below. A potential difference V is applied across all three.



Work through question 2 in order to obtain the value, in terms of *C*1, *C*2 and *C*3, of a single capacitor *C* that would replace all three.

2. (a) The diagram above shows that each capacitor has an identical charge.

Explain how this comes about.

(b) Write down expressions for the following

(i) *V*1 in terms of *Q* and *C*1

(ii) *V*2 in terms of *Q* and *C*2

(iii) *V*3 in terms of *Q* and *C*3

(iv) *V* in terms of *Q* and *C*

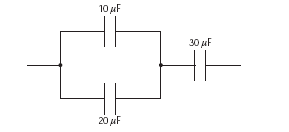
(c) What is the total potential difference *V* across all three capacitors in terms of *V*1, *V*2 and *V*3?

(d) Using your answers to (b) and (c), obtain an expression for *C* in terms of *C*1, *C*2 and *C*3.

You should have noticed that the expressions for capacitors joined in series and parallel are ‘the other way round’ to the expressions for resistors. Resistors in series are simply added together; for capacitors in series you need to add reciprocals and find the reciprocal of the result. Capacitors in parallel add together, whereas for resistors in parallel it is the reciprocals that must be added.

3. What is the combined capacitance of a 10 F capacitor, a 20 F capacitor and a 30 F capacitor connected (a) in parallel (b) in series?

4. What is the combined capacitance of a 10 F capacitor and a 20 F capacitor connected in parallel, and then connected in series to a 30 F capacitor as shown below.



Calculating energy stored

Having seen that the energy depends on the voltage, there are several approaches which lead to the relationship for the energy stored.

Remember! **joules = coulombs × volts’**.

The simplest argument is that with a pd V, a capacitor C will store charge Q, but the energy stored is not Q × V. Why not?

Charge Q

**Q**

**V**

Potential difference

At first, it is easy to push charge on to the capacitor, as there is no charge there to repel it. As the charge stored increases, there is more repulsion and it is harder (more work must be done) to push the next lot of charge on.

A first try says that the pd was on average V/2, so the energy transformed was Q × V/2.

A more general approach says that in moving the charge Q, the pd does not change significantly, so the energy transformed is V × Q. But this is just the area of the narrow strip, so the total energy will be the triangular area under the graph.

i.e. Energy stored in the capacitor = 1/2 QV = 1/2 CV2 = 1/2 Q2/C

(You could show this by integration.)

Worked example: Energy stored

A 10 F capacitor is charged to 20 V. How much energy is stored?

Which equation?

Energy stored =

calculate the energy is stored at 10 V (i.e. at half the voltage).

Compare these values

Why is this the case?

Energy stored in capacitors.

What to do

Work your way through the questions, preferably in the order in which they appear. The first few questions will give you some practice in using the equations relating to energy storage.

Questions

A 50 microfarad (F) capacitor is charged to a pd of 60 V.

1. Calculate the charge on the capacitor.

2. Calculate the energy stored.

3. Calculate the energy stored when the pd is doubled to 120 V.

4. Compare your answers to questions 2 and 3. What does this tell you about the relationship between the energy stored by a capacitor and the pd to which it has been charged?

5. A 1000 F capacitor is charged so that its stores 2.0 J of energy. Calculate the pd to which it has been charged.

6. The incomplete table below contains values of capacitance, charge, pd and energy for a series of charged capacitors. Carry out calculations and fill in the blanks in the table.

| **Capacitance** | **Charge** | **Potential difference** | **Energy** |
| --- | --- | --- | --- |
| 1000 F |  | 16 V |  |
| 10 mF | 0.01 C |  |  |
| 1.0 F |  |  | 100 J |
|  | 2.0 mC | 5000 V |  |
|  |  | 100 V | 50 mJ |
| 33 000 F |  |  | 2.0 J |

A 1.0 F capacitor is charged to a pd of 10 V.

7. Calculate the charge on the capacitor.

8. How much charge flowed through the battery during charging?

9. How many electrons flowed through the battery during charging?

10. Calculate the energy stored by the capacitor.

11. How much energy was transferred from the battery during the charging process?

12. (Rather harder) You should have found different answers for questions 10 and 11. Explain this difference.  
  
  
  
Hints

1. You will have to do quite a bit of rearranging of equations.

6. Think of the definition of potential difference when you are trying to work out how much energy is involved in passing charge through a battery.