Exponential and logarithmic equations

# The exponential decay equation

Explain that the equation *N* = *N*0 e-*t* can be used to generate an exponential decay graph. Work through a numerical example, perhaps related to the dice-throwing analogue (*N*0 = 100;  = 1/6). Make sure that your students know how to use the e*x* key on their calculators.

Emphasise that similar equations apply to activity *A* (*A* = *A*0 e-*t*) and count rate *C* (*C* = *C*0 e-*t*).

(Not all the radiation emitted in all directions by a source will collected by a detector lined up in one direction from a source)

The logarithmic form of the equation

a straight line graph is usually more useful than a curve, particularly when dealing with experimental data.

the equation ln*N* = ln*N*0 - *t*.

Draw a sketch graph to show this relationship

What does the gradient represent?

What does the y intercept represent?

Practice calculations

Fractions and activity

1. In exponential decay the number of atoms remaining after a given interval of time is always the same fraction of the number present at the beginning of the time interval. Therefore, someone might say, however long you wait there will always be some left. A sample of radioactive atoms will last for an infinite time.

Is this reasonable? Explain your answer.

2. The decay constant for caesium-137 is 7.3  10–10 s–1.

Calculate the number of atoms in a sample of caesium-137 that has an activity of

2.0  105 Bq.

Experimental exponentials

No ordinary place on the Earth’s surface is free from ‘background radiation’. In making a measurement of this background a student set up a GM tube and counter and recorded the counter reading every 30 s.

| **Time / s** | **Count** |
| --- | --- |
| 0 | 0 |
| 30 | 13 |
| 60 | 31 |
| 90 | 48 |
| 120 | 60 |
| 150 | 75 |
| 180 | 88 |
| 210 | 102 |
| 240 | 119 |

3. Using these data, determine the background count per minute. What would you expect the total count to be after a further 30 seconds?

A radioactive source is set up in front of the same GM tube in the same laboratory. Counter readings of the activity of the source are taken for 1 minute at time intervals of 1 hour.

| **Time / h** | **A / counts min–1** |
| --- | --- |
| 0 | 828 |
| 1 | 510 |
| 2 | 320 |
| 3 | 202 |
| 4 | 135 |
| 5 | 95 |
| 6 | 70 |
| 7 | 51 |
| 8 | 41 |
| 9 | 38 |
| 10 | 37 |
| 11 | 31 |
| 12 | 33 |
| 13 | 34 |
| 14 | 29 |
| 15 | 35 |

4. What action should be taken to deal with the background count?

5. Plot a graph to show how the activity of the radioactive substance changes with time and derive from it three different values of the half-life.

Calculate the mean of these.

6. Worried by the obvious fluctuations in the final hours of the count, one student suggests that it might be wise to continue counting for at least as long again.   
 Is this a good idea? Explain your answer.

7. There is a slight increase in the count rate towards the end of the experiment.

Is this significant? Explain your answer.

8. Suggest steps you might take to improve the experiment.

Radioactive decay with exponentials

1. The half-life of one radioactive isotope of sodium is 2.6 years. Show that its decay constant is 8.4  10–9 s–1.

2. Calculate the activity of a sample containing one mole of the sodium. (One mole contains 6.02  1023 atoms.)

A scientist wishes to find the age of a sample of rock. Realising that it contains radioactive potassium, which decays to give a stable form of argon, the scientist started by making the following measurements:

decay rate of the potassium in the sample = 0.16 Bq

mass of potassium in the sample = 0.6  10–6 g

mass of argon in the sample = 4.2  10–6 g

3. The molar mass of the potassium is 40 g. Show that the decay constant  for potassium is 1.8  10–17 s–1 and its half life is 1.2  109 years.

4. Calculate the age of the rock, assuming that originally there was no argon in the sample and the total mass has not changed. Show the steps in your calculation.

5. Identify and explain a difficulty involved in measuring the decay rate of 0.16 Bq given above.

6. Iodine 124, which is used in medical diagnosis, has a half-life of 4.2 days. Estimate the fraction remaining after 10 days.

7. Explain how you would find the half-life of a substance when it is known to be more than 10 000 years. Assume that a sample of the substance can be isolated.

In an experiment to find the half-life of zinc-63, a sample containing a sample of the radioactive zinc was placed close to a GM tube and the following readings were recorded. The background count rate was 30 min–1.

| **Time / hours** | **Counts**  **/ min–1** |
| --- | --- |
| 0 | 259 |
| 0.5 | 158 |
| 1.0 | 101 |
| 1.5 | 76 |
| 2.0 | 56 |
| 2.5 | 49 |
| 3.0 | 37 |

8. Plot a graph of count rate against time and use this to find the average time for the count rate to fall to one-half of its previous value.

9. Plot a second graph, ln (count rate) against time, and use it to find the half-life.

10. Discuss which method, 8 or 9, provides a more reliable value.

Radio Carbon Dating

(Diagram: resourcefulphysics.org)

Radioactive decay used as a clock

Because of the predictability of the random behaviour of large numbers of atoms, activity can be used as a clock.



Further reading -Two important dating techniques

Radiocarbon dating

This technique was devised by American physicist W. F. Libby in 1949.

Carbon-14 (14C) is a naturally occurring radioactive isotope of carbon which is produced when neutrons associated with cosmic rays collide with nitrogen (14N) in the atmosphere. The 14C is taken up by living organisms (plants, animals); the ratio of 14C to 12C in living tissue remains constant during the life of the organism and depends only on the relative proportions of 14C and 12C present in the atmosphere. (12C is the `ordinary', stable, abundant isotope of carbon.)

When the organism dies, the ratio of 14C to 12C decreases because the 14C decays to produce nitrogen:



After about 5700 years (the half-life of 14C), the ratio of 14C to 12C falls to half its initial value; after a further 5700 years the ratio halves again ± and so on. The 14C /12C ratio therefore decreases in a known and predictable way as the sample ages. If the original ratio of 14C /12C is known for an organic sample, say bone, and the present ratio can be measured, then an accurate value for the age of the sample can be calculated.

To measure the ratio of 14C to 12C, the sample needs to be ground up, thus destroyed. The 14C in a sample is detected and measured via its radioactive emission, and the concentration of carbon is measured chemically. The method works best on samples containing large quantities of carbon. Thus wood, charcoal, bone and shells of land and sea animals are good archaeological samples.

Around 100 g of wood or 30 g of charcoal is required to obtain a date. Bone contains a smaller proportion of carbon so more bone is required than wood (around 1 kg). Around 100 g of shell is required for dating. The precision of the age measurement decreases with age of the sample because the amount of 14C decreases with time. The half-life of 14C is 5730 ± 40 years so the method is most reliable for dating samples no more than a few thousand years old. For a sample about 50 000 years old the uncertainty is about ± 2000 years. The range of radiocarbon dating is at most 100 000 years.

The method rests on the assumption that the sample gains no 14C after death, and that the atmospheric ratio of 14C /12C has remained constant with time. It is therefore necessary to compare radiocarbon dates against other independent methods of dating such as dendrochronology.

For an article on the radiocarbon method, see:

Radioactive decay. Dobson, K., Physics Review, Vol. 4, No. 2, pp. 18-21

A good World Wide Web site for details on the method and much besides is: <http://www.c14dating.com/>

Dendrochronology

The name of this method comes from the Greek `dendron' (tree) and `chronos' (time).

The cross-section of a tree trunk shows a series of concentric circles known as growth rings. Each growth ring represents one year of the life of the tree. The thickness of each ring depends on the climatic conditions in the year that the ring was laid down. Thus the rings show a distinctive pattern of varying thickness. The same pattern can be seen in timber of different ages and so a pattern covering a considerable period of time can be constructed.

Once the pattern has been established and tied to known dates, a pattern in a timber of unknown date can be matched against the established pattern and its date determined. The first person to suggest dendrochronology as a dating method was Thomas Jefferson, one of the first American Presidents, who suggested its use in dating Native American burial mounds.

The method can be used with wood grown since the last Ice Age, about 10 000 years ago. One of the longest patterns established is that using the Bristlecone Pine which grows in California. The arid conditions of the area allow samples of the pine to survive in good condition for thousands of years.

Apart from its use as an absolute method of dating wood, dendrochronology is also used to calibrate radiocarbon dates. When radiocarbon dates are matched against the tree ring dates, it

is found that radiocarbon `years' do not equate directly with calendar years because the amount of 14C in the atmosphere varies slightly. In general, radiocarbon dating gives a young age for older samples (e.g. a radiocarbon date of 4100 BC may be closer to a real date of 5000 BC).

The method does have its limitations. Climatic conditions vary from place to place and so a particular pattern can only be used locally. The wood has to be in a good state of preservation; such wood is difficult to find on archaeological sites. The wood on a site may not indicate the true age of the site. For example, the wood could have been cut down many years before its use on a particular site. And not all types of wood can be used since some types have growth rings of uniform thickness regardless of climatic condition